3801. Sketch the region, which is a sector. Then use the standard arc length formula $l = r\theta$.

3802. Reflection in $y = x + k$ can be seen as reflection in $y = x$ following by a translation. Consider e.g. $k = 3$ to work out what the translation is.

3803. No implication links these two statements. Find a counterexample against each direction. Against $\implies$, consider a factor of $(x - \beta)^4$. Against $\impliedby$, consider a constant of integration.

3804. Find the acceleration a in terms of t. Then, sketch $|a|$ against t. Solving for $|a| = 21$ should give three t values: use your sketch to determine the correct regions.

3805. Set up the identity

$$4y^4 - 8y^2 \equiv a(by)^4 - a(by)^2.$$

Equate coefficients to find a and b.

3806. Place the $m \times n$ lattice in the positive quadrant, with the lower-left corner at $(0, 0)$ and the upper-right at (m, n). Assume, for a contradiction, that the line $y = \frac{n}{m}x$ does not pass through any lattice points other than $(0, 0)$ and (m, n), and also that $\text{hcf}(m, n) = k$, where $k > 1$.

3807. (a) Differentiate implicitly with respect to e.g. x.
(b) Note that, as $x \to \pm\infty$, the curve approaches $x^2 + y^3 = 0$, which can be analysed explicitly.

3808. Firstly, multiply top and bottom of the integrand by e^x. Then, let $u = e^x + 2$. You'll need partial fractions.

3809. Find the stationary point of $y = x \ln x$. Use this to sketch $y = x \ln x$ and find the range of f. Hence, find the largest domain and codomain over which the function is one-to-one.

3810. Place the three base vertices V_1, V_2, V_3 on an (x, y) plane at $(\cos\theta, \sin\theta)$, for $\theta = 0, 120, 240°$, forming an equilateral triangle with side length $\sqrt{3}$. The fourth vertex is then on the z axis. Then consider the triangle V_1OV_4 in an (x, z) plane.

3811. Find the stationary points, and so the equation of the tangent to the curve at the higher of the two. This forms the surface of the deepest pool that can form. Find its re-intersection with the curve, and set up a single definite integral.

3812. Use the binomial expansion and simplify both top and bottom. You are looking for a factor of h to cancel, so that you can take the limit.

3813. Consider the symmetry of the problem in the line $y = x$. Then look for double roots in the equations for pairwise intersections.

3814. Think graphically. Consider the quantity Q as the area of a region of width 1 in the (x, y) plane. Where should such a region be placed in order to minimise its area? No algebra is required.

3815. Consider the horizontals, verticals and diagonals.

3816. Using the fact that the probabilities of each pair of branches must sum to 1, find q and then p. Then use the conditional probability formula.

3817. (a) Rearrange the proposed relationship to make y/x the subject.
(b) Find the equation of the straight line of best fit in the form $y/x = mx + c$. Then rearrange.

3818. Subtract one equation from the other, and both x^2 and y^2 will cancel, leaving an equation in z.

3819. Write the information given into an equation in p, and solve.

3820. Find the second derivative. Then explain, using a symmetry argument, why it is zero at $x = 0$ but doesn't change sign there.

3821. Consider a possibility space of ${}^8\text{C}_3$ sets of three vertices. To form a triangle congruent to the one shown, you need two vertices on an edge and one on the diagonally opposite edge.

— Alternative Method —

The triangle has side lengths $\left(1, \sqrt{2}, \sqrt{3}\right)$. Choose the first vertex wlog. For success, the next vertex could be a distance $1, \sqrt{2}$, or $\sqrt{3}$ from the first. Find the probabilities of these. Then work case by case: "With the first two vertices a distance 1 apart, how many successful locations are there for the third vertex?"

3822. Show that $x \ln x$ is both positive and concave for $x \in (1, 2)$.

3823. Find and simplify the gradient in terms of p. Then set y to zero in the standard equation of a line $y - y_1 = m(x - x_1)$.

3824. If you need to, use a concrete example, e.g. $P = 28$ to get your bearings.
(a) d_1 must be 1.
(b) Consider the fact that each factor greater than 1 must have a partner in the list of factors. For example, $d_2 \times d_n = P$.

(c) The full list of divisors is $1, d_2, d_3, ..., d_n, P$. Write out the sum you need to calculate, and take out a factor of $\frac{1}{P}$. Then use the results of part (a) to simplify.

3825. The possibility space consists of all sets of three digits $\{1, 2, 3\}$ adding to 5. There are six: three of $(1, 1, 3)$ and three of $(1, 2, 2)$.

3826. Consider the even symmetry of the curve and its behaviour as $x \to \pm\infty$. Also, find $\frac{dy}{dx}$ and look for SPs. Also, look for axis intercepts.

3827. Use the area formula $\frac{1}{2}bh$. The perpendicular heights are easy, because the octagon's angles are (multiples of) 45°. You can do this explicitly, or spot that the perpendicular heights of the smaller triangles add directly to the perpendicular height of the larger triangle.

3828. Differentiate and set up the equation for SPs. Then use the Newton-Raphson method.

3829. Compare the ranges of functions.

3830. (a) Factorise the RHS and look for double roots.
(b) Differentiate with respect to x and look for an undefined gradient.

3831. (a) Use the first Pythagorean trig identity.
(b) Differentiate and use Pythagoras.
(c) Differentiate again.

3832. Call the half-interior angles α at A, β at B and so on. Then $\alpha + \beta + \gamma + \delta = 180°$. Write the sum of a pair of opposite interior angles of $PQRS$ in terms of $\alpha, \beta, \gamma, \delta$, and simplify algebraically to 180°.

3833. This is true. Construct an example generalising, to n dimensions, two equivalent lines $x + y = 0$ and $2x + 2y = 0$ in 2D.

3834. Factorise the denominator of the LHS, and use the first Pythagorean trig identity. Then simplify with a double-angle formula.

3835. In both cases, factorise fully and consider the roots and the multiplicity of the roots. Also, solve the equation for intersections $x^5 - x = x^7 - x$, likewise considering the nature of its roots.

3836. (a) Show that the stationary point must be a triple root.
(b) Differentiate twice and solve a quadratic.
(c) Find the relevant point of inflection, and use the x value you get to factorise the quartic.

3837. 11! is the number of anagrams if all the letters are considered distinguishable. Divide this by three overcounting factors: these are the numbers of ways of ordering the repeated letters amongst themselves.

3838. Find the time of flight in terms of initial speed u and angle of projection θ. Use this time to set up expressions for the range and the maximum height. Then form an equation and solve for θ.

3839. Factorise the top as a difference of two squares.

3840. (a) Complete the square in y.
(b) Use (a).
(c) Find the intersections with the line $x = 3$, and set up a definite integral with respect to y.

3841. One approach is algebraic: you can assume, wlog, one circle to be $x^2 + y^2 = 1$ and the other to be $x^2 + (y - k)^2 = r^2$. Solve for intersections.

— Alternative Method —

Another is graphical: assume that two circles are tangent at distinct points. Show, using tangents and radii, that the centres must coincide.

3842. If an iteration $x_{n+1} = g(x_n)$ is to produce period 2 from $x_0 = \alpha$, then α must be a root of the equation $x = g^2(x)$.

3843. Sketch the graphs, calculating the coordinates of the vertices of the resulting quadrilateral.

3844. (a) Consider rotational equilibrium around the centre of the sphere.
(b) Consider the boundary case: the reaction force between the block and the sphere is equal to zero, but the two are still tangent.

3845. Consider the unit square in the (x, y) plane as the possibility space. Find the area of the successful region.

3846. (a) Find $\frac{dy}{dx}$ with the parametric differentiation formula for the first derivative. Simplify this, then substitute it into the given formula.
(b) Consider which polynomials have constant derivatives.

3847. Factorise the quadratic, so that you have a pair of linear factors, each raised to the power of 6. This gives a polynomial graph of order 12 (even degree) with two sextuple roots.

3848. Expand the brackets, inside the sum, then split the sum up over the four terms.

3849. (a) Resolve along the string to find acceleration during the first second. Use *suvat*s to find the position and velocity of one bob (and thus by symmetry the other) at $t = 1$.

(b) Use the values calculated as initial conditions for (relative) projectile motion, looking for the difference in height to be 10 metres. (Since the pulley is small, horizontal displacement can be neglected.)

You can, in fact, ignore the acceleration, since both bobs, when in projectile motion, have the same acceleration g ms^{-2}.

3850. Solve for intersections with $x+y=0$, and factorise the resulting equation to find two distinct double roots. Alternatively, you could use calculus.

3851. First, find the equation of the tangent to $y = \sin x$, where x is measured in radians. Then consider the scale factor of the stretch in the x direction which converts the one sine graph to the other. Apply this to the tangent.

3852. The t-period of the curve is the lcm of the periods of its x and y components.

3853. Let $u = \sqrt{x}$. Then find $2\,du$ in terms of x and dx; you can substitute this equation directly when enacting the substitution.

3854. Consider a^{LHS} and a^{RHS}.

3855. Consider a discontinuous function whose graph has a vertical asymptote.

3856. (a) Write L as $y = m(x-1)+1$ and substitute into the equation of C.

(b) Take out a factor of $(x-1)$, corresponding to the point $(1, 1)$, and set the discriminant of the remaining quadratic to zero.

3857. Draw a tree diagram to visualise the possibility space. You are looking to find $\mathbb{P}(G \mid R)$. Restrict the possibility space to rejected claims.

3858. (a) Find the cross-sectional area of the water in the ditch using integration. You need to be careful what x stands for, because it could play two different roles: the boundary value of a definite integral, and also the variable of that integration. Call the boundary value x and the variable of integration p.

(b) Show that the surface area is proportional to x, and hence to $V^{\frac{1}{3}}$. Set up a DE for y modelling the evaporation, and solve it.

3859. Use a double-angle formula on $\sin 4\theta$, but not on $\sin 2\theta$ or $\cos 2\theta$. Then factorise. This will produce equations for $\sin 2\theta$ and $\cos 2\theta$. The former has four roots in the domain, the latter three.

3860. Assume, for a contradiction, that an equilateral triangle has integer values for both its perimeter P and area A. Set up an equation linking P and A. Show that, according to the assumption, $\sqrt{3}$ is rational.

3861. (a) Rearrange to make y the subject. Differentiate explicitly.

(b) Note that your rearrangement in part (a) has introduced new points to the graph.

3862. Calculate derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$ using the product rule. Then substitute in the parametric equations. This should give the RHSs of the coupled DEs.

3863. To transform the vertex, you need to reflect the curve in both x and y axes.

3864. (a) Draw a radius to the point at which the rope leaves the barrel. Use the fact that tangents to a point are symmetrical to set up congruent triangles. You'll then need to use a double-angle formula.

(b) Include two tensions, the weight and reaction from the deck.

(c) Resolve vertically.

3865. (a) Find $f'(x)$ and explain why zero cannot be in its domain.

(b) Solve $f'(x) = 0$. Simplify to $5x^{\frac{4}{5}} - 6x^{\frac{2}{5}} + 1 = 0$. This is a quadratic in $x^{\frac{2}{5}}$. You should get four values of x satisfying $f'(x) = 0$. Test f at these values.

— Alternative Method —

Factorise $f(x)$ as a quintic in $x^{\frac{1}{5}}$ and look for double roots.

3866. For the particle to reverse its direction of travel, both components of velocity $\dot{x}$ and $\dot{y}$ must drop to zero.

3867. Divide the equation by ab. Then consider the boundary equation as a unit circle transformed by (different) stretches in the x and y directions.

3868. (a) The graph is a positive cubic with single roots at $x = a, b, c$.

(b) The graph is a positive sextic with double roots at $x = a, b, c$.

(c) The graph is a positive nonic with triple roots at $x = a, b, c$.

3869. Let $y = \arccos x$. So, $x = \cos y$. Differentiate this with respect to x. Then rearrange and use the first Pythagorean trig identity.

3870. Substitute the value $x = 0.001$. Then simplify and rearrange, using the fact that $\sqrt[3]{0.027} = 0.3$.

3871. (a) Find the derivatives and substitute in.
(b) Differentiate the solution to find $\dot{x}$. Substitute in $t = 1$ and $t = 5$. The constant A should then cancel when you consider the percentage change.

3872. Write $\sin 3t \equiv \sin(2t+t)$, and then expand using a compound-angle formula. Then use double-angle formulae to write the resulting expression in terms of $\sin t$, and substitute for y.

3873. (a) Use the conditional probability formula.
(b) Find $\mathbb{P}(A)$ in terms of k using the conditional probability formula. Hence, find $\mathbb{P}(A \cap B')$. Use the same kind of argument for $\mathbb{P}(A' \cap B)$. Subtract the three probabilities from 1 to find $\mathbb{P}(A' \cap B')$.
(c) $\mathbb{P}(A \cup B)$ is bounded below by 0 and above by 1. One of these bounds is attainable, the other is not.

3874. The axis intercepts are $(0, 1)$ and $(1, 0)$. So, the line has equation $y = 1 - x$. The equation to solve, therefore, is

$$1 - x = \left(1 - \sqrt{x}\right)^3.$$

Substitute $z = \sqrt{x}$ and expand binomially.

3875. (a) Consider the shape of a quartic.
(b) Use the Newton-Raphson method.
(c) Take out a squared factor. The factor which remains should be irreducible.

3876. Use the first Pythagorean trig identity. You should justify taking the positive square root. Then use a small-angle approximation.

3877. To produce a positive product, the factors must both be positive or both be negative. Consider (sketch) the behaviour of $x \mapsto x - |x|$.

3878. Set $\frac{dx}{dt} = 0$ and solve.

3879. Consider intersections with the line $y = x$.

3880. Let $u_1 = a$ and $u_2 = b$. Express $u_3, u_4, u_5, \ldots$ in terms of a and b, simplifying as you go. You should find that $u_7 = a$ and $u_8 = b$.

3881. Write the integrand in partial fractions.

3882. (a) Solve the equation $\cos y = -\cos x$ with the identity $-\cos x \equiv \cos(\pi - x)$.
(b) Consider the general case as a transformation of the special case in (a).

3883. Simplify each side individually, noting that the cross-terms are $e^x \cdot e^{-x} = 1$.

3884. Take out a factor of $(x - y)$. The remaining factor is quadratic, giving the set of points as a straight line and a parabola.

3885. Choose $u = \ln x$, because $\ln x$ is the factor which gets significantly simpler when differentiated.

3886. Sketch the scenario carefully. Find the equation of the tangent to $y = x^2$ at $\left(p, p^2\right)$.

3887. Use log rules on the middle term, then factorise the LHS into the form $(\log_2 x + a)(\log_2 x + b)$.

3888. The graph is polynomial, so the only symmetries it could possibly have are as follows:

① rotational symmetry of order 2,
② reflective symmetry in a line $x = k$.

Rule ① out using the degree of the polynomial. Rule ② out by considering the multiplicity of the roots.

3889. Each of the curves has period π. Solve for intersections in $[0, \pi)$ using the second Pythagorean identity. Then set up a definite integral for the area enclosed by the curves. Use two standard integral results.

3890. (a) Tilt the toy at e.g. 15°. There are four forces: P, the weight, reaction and friction. The line of action of the reaction passes through O.
(b) Horizontal equilibrium gives the friction as equal to P. Take moments about O and find F, thereby finding P.

3891. The derivative $f'(x)$ is a quadratic with roots at $x = \pm 2$. Use this to construct $f'(x)$ algebraically. Then differentiate $f'(x)$ and analyse $f''(x)$.

3892. (a) A Venn diagram might help.
(b) Consider the conditional probability formula.
(c) Again, a Venn diagram might help.

3893. Find the intersections of the curves in terms of n, and set up a single definite integral for the area enclosed.

3894. (a) Use $\sin 2x \equiv 2\sin x \cos x$. Then divide top and bottom of the fraction by $\cos^2 x$.

(b) Use the second Pythagorean trig identity to write top and bottom as quadratics in $\tan x$. Factorise these, and take the positive square root.

3895. From the partial sums, write equations in n and the common difference d. Solve simultaneously.

3896. Rearrange to LHS $= 0$ and factorise.

3897. Spotting that $x = -1$ is a root, take out a factor of $(x + 1)$. Then, since the quadratic factors must be monic and the constant terms must multiply to 1, you are looking to factorise the quartic into the form

$$(x^2 + ax + 1)(x^2 + bx + 1).$$

Equate coefficients of x^3 and x^2 to produce a pair of simultaneous equations in a and b.

3898. Write the improper algebraic fraction as the sum of a polynomial and a proper fraction. Then you can integrate by inspection.

3899. In order to be able to construct a quadrilateral, it is sufficient that each length is individually smaller than the sum of the other three.

3900. Consider all possible outcomes $\{2, 3, ..., 12\}$ for the sum on two dice. Calculate the probabilities of these (there is a simple pattern), then square the probabilities and add them.

——— End of 39th Hundred ———